USC International Accelerator Program

QED Diagnostic Math Evaluation for (Non-Software) Engineering Students

The problems below cover some of the basic topics covered in a typical 3 sequence course in calculus and an introductory differential equations course. A student preparing for a Master's degree in Engineering at University of South Carolina should have minimum difficulty with the 20 questions below. The correct answers are provided with a diagnostics legend that explains possible outcomes based on your score.

Evaluate:

1. $\lim_{x\to 0} \frac{\sin x}{x}$
2. $\lim_{x\to \infty} \frac{3x}{2x-1}$
3. $\int \frac{x+1}{x(x^2+1)}\,dx$
4. $\int \ln x\,dx$
5. $\int \sqrt{1-x^2}\,dx$
6. *Find the radius and interval of convergence for the series* $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$
7. *Find Taylor series expansion for* $f(x) = sinx$ *(with center at 0)*
8. *Find the Maclaurin series and radius of convergence for* $f(x) = sin(x^2)$.
9. $Find\ the\ derivative\ y'\ if\ \ y = x\,arctan(2x).$
10. $Find\ y'\ if\ \ y = (x^2 + x^3)^4.$
11. $Solve\ the\ differential\ equation: xy' + 2y = \frac{cos\,x}{x};\quad y(\pi) = 0,\quad x > 0.$
12. $Solve: y' = \frac{2x}{y+x^2y};\quad y(0) = -2.$
13. $Solve: y'' - 2y' - 3y = 0;\ y(0) = 2,\ y'(0) = 2.$
14. $Find\ the\ general\ solution\ for\ the\ differential\ equation:\ y'' + 4y = e^{3x}.$
15. $Find\ the\ general\ solution\ for\ the\ differential\ equation: y'' - 3y' = sin\,2x.$
16. *Find three positive numbers whose sum is 100 and whose product is a maximum.*
17. *Find the local maximum and minimum values of the function* $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2.$
18. $If\ \boldsymbol{F}(x,y) = 2xcosy\ \boldsymbol{i} - x^2siny\ \boldsymbol{j}, find\ f\ such\ that\ \nabla \mathbf{f} = \mathbf{F}.$
19. $Find\ the\ equation\ of\ the\ plane\ though\ the\ point\ (0,0,0)\ orthogonal\ to\ the\ vector\ \boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}.$
20. *Find the volume of the region bounded by the paraboloid* $z = x^2 + y^2$ *and below the triangle enclosed by the lines* $y = x, x = 0, and\ x + y = 2$ *in the* $xy - plane.$

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Answers:

1. 1
2. $\frac{3}{2}$
3. $\ln|x| + arctanx + C$
4. $xlnx - x + C$
5. $\frac{1}{2}\left(arcsinx + x\sqrt{1-x^2}\right) + C$
6. $Radius = 1;\ Interval\ [2,4)$
7. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$
8. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!} = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots ;\ R = 1$
9. $y' = arctan2x + \frac{2x}{1+4x^2}$
10. $y' = 4(x^2 + x^3)^3(2x + 3x^2)$
11. $y(x) = \frac{sinx}{x^2}$
12. $y(x) = -(2(\ln(1+x^2) + 2))^{\frac{1}{2}}$
13. $y(x) = e^{-x} + e^{3x}$
14. $y(x) = C_1 Sin2x + C_2 Cos2x + \left(\frac{1}{13}\right)e^{3x}$
15. $y(x) = C_1 + C_2 e^{3x} - \left(\frac{1}{13}\right)Sin2x + \left(\frac{3}{26}\right)Cos2x$
16. $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$
17. $Local\ maximum\ f(0,0) = 2;\ local\ minimum\ f(0,4) = -30$
18. $f(x,y) = x^2 cosy$
19. $x - 2y + 5z = 0$
20. $\frac{4}{3}\ cubic\ units$

Score	Comment
Between 16 and 20 correct	You have demonstrated the minimum proficiency necessary to satisfy the prerequisites for select USC Engineering programs.
Less than 16 correct	
	You have not demonstrated the minimum proficiency necessary to satisfy the prerequisites required for select USC Engineering programs. You must not check the box on the application in the item entitled Proficiency Declaration