

# USC International Accelerator Program

## QED Diagnostic Math Evaluation for Chemical Engineering Students

The problems below cover some of the basic topics covered in a typical 3 sequence course in calculus, an introductory differential equations and linear algebra course. A student preparing for a Master's degree in Chemical Engineering at University of South Carolina should have minimum difficulty with the 20 questions below. The correct answers are provided with a diagnostics legend that explains possible outcomes based on your score.

Evaluate:

1.  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
2.  $\lim_{x \rightarrow \infty} \frac{3x}{2x-1}$
3.  $\int \frac{x+1}{x(x^2+1)} dx$
4.  $\int \ln x dx$
5.  $\int \sqrt{1-x^2} dx$
6. Find the radius and interval of convergence for the series  $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$
7. Find Taylor series expansion for  $f(x) = \sin x$  (with center at 0)
8. Find the Maclaurin series and radius of convergence for  $f(x) = \sin(x^2)$ .
9. Find the derivative  $y'$  if  $y = x \arctan(2x)$ .
10. Find the determinant of the matrix  $A = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 3 & 4 \end{vmatrix}$
11. Solve the differential equation:  $xy' + 2y = \frac{\cos x}{x}$ ;  $y(\pi) = 0$ ,  $x > 0$ .
12. Solve:  $y' = \frac{2x}{y+x^2y}$ ;  $y(0) = -2$ .
13. Solve:  $y'' - 2y' - 3y = 0$ ;  $y(0) = 2$ ,  $y'(0) = 2$ .
14. Find the general solution for the differential equation:  $y'' + 4y = e^{3x}$ .
15. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$ , find  $X$  if  $AX = B$ .
16. Find the inverse of the matrix  $\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
17. Find the local maximum and minimum values of the function  $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$ .
18. If  $F(x, y) = 2x \cos y \mathbf{i} - x^2 \sin y \mathbf{j}$ , find  $f$  such that  $\nabla f = F$ .
19. Find the equation of the plane through the point  $(0,0,0)$  orthogonal to the vector  $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ .
20. Find the volume of the region bounded by the paraboloid  $z = x^2 + y^2$  and below the triangle enclosed by the lines  $y = x$ ,  $x = 0$ , and  $x + y = 2$  in the  $xy$ -plane.

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## Answers:

1. 1
2.  $\frac{3}{2}$
3.  $\ln|x| + \arctan x + C$
4.  $x \ln x - x + C$
5.  $\frac{1}{2}(\arcsin x + x\sqrt{1-x^2}) + C$
6. Radius = 1; Interval [2, 4)
7.  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
8.  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!} = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots; R = 1$
9.  $y' = \arctan 2x + \frac{2x}{1+4x^2}$
10. -40
11.  $y(x) = \frac{\sin x}{x^2}$
12.  $y(x) = -(2(\ln(1+x^2) + 2))^{\frac{1}{2}}$
13.  $y(x) = e^{-x} + e^{3x}$
14.  $y(x) = C_1 \sin 2x + C_2 \cos 2x + \left(\frac{1}{13}\right)e^{3x}$
15.  $X = \begin{bmatrix} 11 \\ -8 \end{bmatrix}$
16.  $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 1 & 2 & -1 \end{bmatrix}$
17. Local maximum  $f(0,0) = 2$ ; local minimum  $f(0,4) = -30$
18.  $f(x,y) = x^2 \cos y$
19.  $x - 2y + 5z = 0$
20.  $\frac{4}{3}$  cubic units

Diagnostic Math

Score	Comment
Between 16 and 20 correct	You have demonstrated the minimum proficiency necessary to satisfy the prerequisites for select USC Engineering programs.
Less than 16 correct	You have not demonstrated the minimum proficiency necessary to satisfy the prerequisites required for select USC Engineering programs. You must not check the box on the application in the Declaration of Math Proficiency.