Design of Advanced Time-Frequency Mutual Information Measures for Aerospace Diagnostics and Prognostics

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Abstract—Accelerometer data has been gathered from accelerated conditioning in grease lubricated and lubrication deprived gear meshes in AH-64 helicopter intermediate and tail rotor gearbox, which are commonly problematic components of the Apache helicopter platform. These tests were performed in a controlled drive-train research test bed, simulating drive-train conditions to improve diagnostic and prognostic capabilities of Condition Based Maintenance (CBM) practices in Integrated Vehicle Health Monitoring System - Health Usage Monitoring System (IVHMS-HUMS) and other comparable CBM packages, as monitored by a standardized Digital Source Collector (DSC) system. Time-frequency representations of vibration measurement collected from two spaced sensors are used to provide signature analysis of transient system harmonics. Furthermore, the time-frequency mutual information advanced signal processing technique is then proposed and validated using vibration data. The measure advances the development of mutual information health indicators to quantify degradation of the helicopter power train. The AH-64 test systems perform under stress in realistic loading conditions and lifetime accelerated aircraft aging is monitored using the proposed advanced signal processing techniques for baseline tests for comparison with faulted conditions.1 2

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1. INTRODUCTION

Continued operational availability of air- and rotorcraft is vital to safety and reliability in the United States Army under conditions of military operations as well as troop transport. In order to properly maintain a diverse fleet of military aircraft at a cost premium, existing methods of time-based maintenance must be supplemented by effective methods of condition based maintenance. In order to monitor the health status of tail rotor systems, a variety of signals are collected, including vibration [1], [2], acoustic [3], and temperature. Over the past decade great advancements have been made in health diagnostics and vibration management in military helicopters [4]. The successes to date have resulted in the large-scale deployment of increasingly useful health monitoring systems such as HUMS (Health and Usage Monitoring Systems) using Vibration Management Enhancement Program (VMEP) hardware, which have generated a wide range of benefits from increased safety to reduced maintenance costs. Most CBM tools such as HUMS for Apache and Blackhawk helicopters assist machinery maintainers in identifying faulted components through the use of simple interfaces and indicators. The most commonly utilized functions are condition indicators (CI), which output a dimensioned or dimensionless single scalar value to monitor key factors most frequently related to frequency analysis of vibration signature. Condition indicators need not be based upon vibration analysis alone and may include component temperatures or acoustic data for separate or fused CIs. Examples of common indicators in machine diagnostics and prognostics include: spectral peak analysis, envelope analysis, energy ratio, crest factor, sideband index, and kurtosis of residual signals [5]. These CI values are typically compared with pre-established thresholds in a simple decision tree classifier which assign the CI some form of ranked class such as “Good,” “Caution,” or “Exceeded,” and these classes are then utilized by maintainers in vital decision-making processes. A given component can have several CIs which may additively form a health indicator (HI). However, CIs or HIs are not fault-specific; multiple fault types can affect the value of a single CI, and a single fault could affect multiple CIs.

While various condition and health indicators do exist, we aim to improve their effectiveness by developing new general methods for fault analysis based around time-frequency analysis that could be used in existing or new CIs for indication of machinery failure [6],[7]. Previous research focused solely on the comparison of forward and aft hanger bearings, in this paper we present enhancements to our mutual information measure and metrics that allow for normalization of hangar bearing and gearbox data with respect to signal energy and comparison between multiple

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Figure 1 – Spectrogram of bearing vibration signature from forward sensor under conditions of shaft misalignment and bearing unbalance.

An advanced time-frequency mutual information measure is proposed to detect and categorize transient changes in vibration harmonic signatures leading to failure of drive-train components such as hanger bearings and gearboxes. This innovative technique is characterized by (1) application of mutual information statistical methods to Cohen’s class time-frequency distributions (2) the extension of Rényi information for complexity assessment from self-information of continuous bivariate distributions to applications in mutual information of closely related signals and (3) in-phase and quadrature subsets of mutual information measure.

2. APPLICATIONS OF TIME-FREQUENCY ANALYSIS AND INFORMATION MEASURE

To explore concepts of a mutual information measure for potential use in sensor fusion, information theory must be addressed concerning time-frequency distributions and analogous concepts in statistics. We consider the concept of signal complexity and its application in time-frequency vibration analysis. These concepts could be further extended to acoustic analysis and additional signals. Intuitively, a signal component is a concentration of energy in some domain (time, frequency, or a combined time-frequency density distribution). However, it is not an easy task to translate complexity of time-frequency distribution into a quantitative measure on time and frequency domain [8], [9].

Instead, in the case of a deterministic signal, it may be useful to find a quantitative measure of signal complexity and information content under the assumption that signals with high complexity must be constructed from increased numbers of elementary components. For example, Figure 1 and Figure 2 show the time-frequency distributions of separated drive-train bearing vibration signatures in conditions of misalignment and bearing unbalance. These bearings experience very different loading in frequencies 5 kHz and over while loading below 5 kHz is similar and less distinct. A measure of self and/or mutual information paired with signal complexity could provide general indication of system status or loading without need for examination of the frequency content.

A promising approach to calculate signal complexity is the use of entropy functions. Given the general Cohen’s class time-frequency distribution defined as a convolution of the Wigner distribution, $W_2(t, \omega)$, and a kernel function, $\Phi(t, \omega)$, we obtain the following expression [13]:

$$C_s(t, \omega; \Phi) = \frac{1}{4\pi^2} \iiint s^*(u - \frac{\tau}{2}) s(u + \frac{\tau}{2}) \Phi(0, \tau) e^{-i(0t - juu) + i\theta} d\theta d\tau du$$

Use of the Cohen’s class distribution permits a more general solution allowing for variable kernel selection. The kernel function of the distribution is described by the $\Phi(t, \omega)$ term in (1). The theory described in this section presents analysis for the general case of the Cohen’s class time-frequency distribution while any distribution kernel could be selected when applying the time-frequency mutual information measure including, but not limited to, the general (Cohen’s class), spectrogram, Zhao-Atlas-Marks, Wigner, Choi-Williams, or a reduced interference distribution (RID) kernel [13]. These kernel functions must still meet certain criteria which will be outlined.

With time- and frequency- marginal properties, we can focus on applying principles of statistical analysis useful for condition based measurements utilizing the Cohen’s class
It would be desirable to implement the concept of Shannon entropy in terms of the Cohen’s class:

$$H(P) = - \iint P(x,y) \log_2(P(x,y)) \, dx \, dy$$  \hspace{1cm} (2)

$$H(C_S) = - \int C_S(t,\omega) \log_2(C_S(t,\omega)) \, dt \, d\omega$$  \hspace{1cm} (3)

as well as the classical mutual information of two random processes, $I(X;Y)$, given by the following equations:

$$I(X;Y) = \iint P(X,Y) \log_2\left(\frac{P(X,Y)}{P(X)P(Y)}\right) \, dx \, dy$$  \hspace{1cm} (4)

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$  \hspace{1cm} (5)

Here (4) and (5) are equivalent expressions and $H(X)$, $H(Y)$, and $H(X,Y)$ are developed from self and mutual information cases of (2).

Unfortunately, the conditions of negativity in many potential time-frequency representation kernel selections prohibit the application of the Shannon entropy due to the logarithmic function. We must be careful to address the process of extending the concepts of self and mutual information to time-frequency distributions in the general case of the Cohen’s class. Thus, in this paper we will utilize the spectrogram of the signal which guarantees positivity of the time-frequency distribution.

To overcome the limitations of strict non-negativity implied by the logarithmic function, Williams, Brown, and Hero proposed a measure of time-frequency self information [9] by use of the generalized Rényi information. The definition of the generalized Rényi information [10] of a continuous bivariate distribution, specifically the Cohen’s class or a joint distribution of two signals, is defined as follows:

$$H_\alpha(C_S) = \frac{1}{1 - \alpha} \log_2 \left( \int \int C_S(t,\omega) \, dt \, d\omega \right)^\alpha$$  \hspace{1cm} (6)

$$I_{S_{1}S_{2}}(t,\omega;\Phi) = \frac{I_{S_{1}S_{2}}(t,\omega)}{\sqrt{C_{S_{1}}(t,\omega)C_{S_{2}}(t,\omega)}}$$  \hspace{1cm} (7)

When sensors with different energy levels are used, such as potential cross-distributions between bearing and gearbox vibration sensors, each signal must be normalized by its own energy at the signal level. The same derivations of [15], [16] still apply with the initial normalization leading to the following expression of the time-frequency based mutual information measure:

$$H(S_1,S_2) = \frac{1}{1 - \alpha} \log_2 \int \int I_{S_{1}S_{2}}(t,\omega;\Phi) \, dt \, d\omega$$  \hspace{1cm} (8)

### 3. Experimental Setup

The CBM center at The University of South Carolina has an AH-64 Helicopter tail rotor drive-train test stand for on-site data collection and analysis [1]. The test stand includes an AC input motor rated at 400 horsepower to provide input drive to the configuration, a multi-shaft drive train supported by hanger bearings, flex couplings at shaft joining points, two gearboxes, and an absorption motor of matching rating to simulate the torque loads that would be applied by the tail rotor blades. The test stand, with labeled picture provided in Figure 3, was used to collect data to be used in conjunction with historic helicopter vibration data to develop the baseline of operation for the systems under test and subsequent gearbox studies. Baseline and low output torque loading profiles are given in Tables 1 and 2. The measurement devices were placed at the forward and aft hanger bearings and both gearboxes, as shown in Figure 3. The intermediate gearbox is pictured in closer detail in Figure 4 with related sensors.
Figure 5 – Baseline comparisons of the mutual information measure (a) Misaligned-Balanced, (b) Aligned-Unbalanced, (c) Misaligned-Unbalanced 4-5 (Exhibits unbalance on 2 drive-shafts) and (d) Misaligned-Unbalanced 3-5 (Exhibits unbalance on 3 drive-shafts), with the Aligned-Balanced Case.

The forward and aft hanger bearing vibration signals are denoted as S\(_1\) and S\(_2\) and the gearbox signals S\(_3\) and S\(_4\) (Figure 3 and Figure 4). The physical separation between accelerometers on the bearings is 1.43 m. All signals from sensors were sampled at 48 kHz. The data is acquired through a standardized Modern Signal Processing Unit (MSPU) with redundant data acquisition systems. For the self and mutual information measures of both bearing and gearbox vibration signatures, similar data organization is used to previous work \[15\] wherein short frames and windows of data are used to subdivide and apply the mutual information measure.

Tables 1 and 2 summarize the experimental setup of baseline tests and low torque gearbox studies in terms of rotational speed, torque, and time duration. The rotational speed ramp-up consists of an intermediate speed ramp to 600 RPM for operator verification of normal operation before a second stage ramp to 4863 RPM. Ramp-down procedure follows the same stages with deceleration from 4863 RPM to 600 RPM, then to a resting state. Output torque is applied from the absorption motor by descriptions provided in Table 2.

Table 1. Loading Profile for 30 Minute Baseline Tests

<table>
<thead>
<tr>
<th>Rotational Speed (RPM)</th>
<th>Output Torque (ft-lb)</th>
<th>Input Torque (ft-lb)</th>
<th>Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramp-up</td>
<td>--</td>
<td>--</td>
<td>3</td>
</tr>
<tr>
<td>4863</td>
<td>0-111</td>
<td>32.35</td>
<td>10</td>
</tr>
<tr>
<td>4863</td>
<td>111-371</td>
<td>108.71</td>
<td>50</td>
</tr>
<tr>
<td>4863</td>
<td>111</td>
<td>32.35</td>
<td>10</td>
</tr>
<tr>
<td>Ramp-down</td>
<td>--</td>
<td>--</td>
<td>7.5</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION

Comparison of Drive-train Hangar Bearings

The mutual information measure discussed in Section 2 and shown in Figure 5 provides a graphical interpretation of monitored condition by analyzing the amount of mutual data shared between two vibration signals (S\(_1\) and S\(_2\) displayed in Figure 5). With normalization of signal energy to satisfy the conditions of the information measure, bearing vibration data and gearbox vibration data can be measured using the updated theory of Section 2. The mutual information measure is comprised of a quadrature component and an in phase component. Figure 5 shows the scatter plot distribution of the in phase component of the measure on the x-axis and the quadrature component of the measure on the y-axis. In the condition of system unbalance, as seen in Figure 5(b), (c), and (d), which compares misaligned and unbalanced experimental settings to the standard baseline, the in-phase component increases in information bits while quadrature measure typically increases at a reduced rate.
Similarly, misalignment can be observed to increase the number of information bits of the mutual measure contained in the quadrature component, as shown in Figure 5 (a) and (d). Additional studies should be analyzed and compared to determine if these trends are truly linear as they appear to be from observation in Figure 5. It would appear that Figure 5 (c) and (d) which were tested under both misalignment and unbalance conditions, as well as combination settings, have differing degrees of misalignment and unbalance yielding different distributions which follow the established trends along the quadrature and in phase components.

A proposed development in condition indicators could focus on the mean values of in phase and quadrature measures between forward and aft hanger bearings (or intermediate and tail rotor gearboxes). In this scheme, one could calculate two scalar values and develop a health indicator such that baseline status in this instance would yield 0.674 in phase mean and 1.340 quadrature mean. The “worst case” test setting of Misaligned-Unbalanced 3-5 pictured in Figure 5 (c) exhibits unbalance on three drive-shafts and shows the highest value of in phase mean at 2.541 (2.425 by quadrature measure). Further options exist to provide summaries of bearing and gearbox mutual information.

**Signature Analysis of Gearbox Vibration**

Under conditions of lifetime simulated aging and grease drought, worst case damage has been emulated on transmission gearbox teeth and mesh. Figure 6 shows the gearbox teeth and gear mesh at the beginning of accelerated worst-case testing and the results after 100 hours.

The first step of gearbox data analysis is the use of the time-frequency distribution, using the Zhao-Atlas Marks kernel function, to identify key transient time-frequency signatures of the gear teeth and gear mesh. This kernel selection provides improved frequency resolution of transients and increases the likelihood of detection of the transient signal in the time-frequency domain.

Provided in Figures 7 and 8 are time-frequency distributions of the intermediate gearbox vibration and tail rotor gearbox signatures. As the tail rotor gearbox reaches its service limit (in this case aided by a lack of lubrication), increased signal content can be seen compared to the signal content of the intermediate gearbox (Figure 7). Some similarities can be drawn from vibration content under 2kHz, however, the vibration content of the tail rotor gearbox clearly shows additional harmonics and transient variation.

**Figure 6 –** Pictures of the actual gear teeth while testing a simulated grease drought after 100 hours of performance before (left) and after 500 hours of operation (right) when the experiment was stopped.

**Figure 7 –** Time-frequency distribution of intermediate gearbox vibration signature.

**Figure 8 –** Time-frequency distribution of tail rotor gearbox vibration signature under grease drought conditions.
Provided in Figures 7 and 8 are time-frequency distributions of the intermediate gearbox vibration and tail rotor gearbox signatures. As the tail rotor gearbox reaches its service limit (in this case aided by a lack of lubrication), increased signal content can be seen compared to the signal content of the intermediate gearbox (Figure 7). Some similarities can be drawn from vibration content in frequency ranging under 2kHz, however, the vibration content of the tail rotor gearbox clearly exhibits additional harmonics and transient variation on the time frequency domain.

We can now consider the self information of each signal to see the utility of the measure as a potential scalar valued condition indicator. The self information of the intermediate gearbox signal is given from (6) as 7.0749 bits in contrast with a self information value of 8.6120 bits reported from analysis of the tail rotor gearbox signal. These results seem intuitive as the signal entropy seems to increase based on visual inspection of the time-frequency distributions. Again, these values can be applied as indicators of holistic machine state as a measure of the general amount of signal and transient harmonics.

**Effects of Speed and Torque in Gearbox Vibration Signatures**

Having previously considered the results of the time-frequency distribution of the gearbox vibration signals and self information, it is necessary to consider the mutual information of the gearbox data. Figure 9 displays the results of the new normalization in the case of a variable output torque. Horizontal and vertical axes represent the in-phase and quadrature sub-sets of the mutual information measure. Here variable torque from 111 to 371 ft·lbf is plotted in comparison to constant torque at 371 ft·lbf. An important consideration to note is that the control program of the test stand measures speed by use of a dedicated tachometer while torque is calculated using a current measurement. Therefore, even in a “constant torque” control regime based on a constant torque reference, some variations are likely to be seen based on current ripple and other factors.

It is obvious from Figure 9 that the mutual information measure of the gearbox vibration signatures is much higher than the respective hanger bearing signatures. This phenomenon could be due to the fact that the gearboxes share much more frequency content than the hangar bearings do when comparing the results of Figures 1 and 2 to those in Figures 7 and 8. Shared frequency content between gearbox signals in Figures 7 and 8 include the mesh frequencies, input and output shaft frequencies and their harmonics. This can be contrasted with the time-frequency plots for bearing analysis given in Figures 1 and 2 which show a disparity of common frequency content at and above 5 kHz. Figure 9 shows the mutual information measure between vibration collected from intermediate gearbox ($S_i(t)$) and tail rotor gearbox ($S_r(t)$).

In Figure 9, the mutual information measure tends to have equal in-phase and quadrature components as well when comparing variable torque to constant torque. The same can be said of the mutual information of gearbox signatures at constant speed as seen in Figure 10. This can be interpreted as a stable relative complexity of instantaneous frequencies in common between the two signals and their instantaneous phase shifts, under the same health status of the mechanical components. However, variation in the speed, as shown in Figure 10, causes scattering of mutual information measure values around the mean value which indicates more complex relative relation between the frequencies in common between the two signals and their relative phase shift.

Also, it appears that the values of the mutual information measure in relation to changing speed (Figure 10) change in a wider range in both in phase and quadrature axes than variation of torque in Figure 9. Under different fault operating conditions, the relative relation between the two components of the mutual information measure changes according to the status of the mechanical parts as shown in Figure 5. Lastly, a number of the low frequency harmonics are closely related to the rotational speed of drive-train components and could complicate the interpretation of these results. Further analysis is underway to determine a more precise relationship between the scattering in the mutual
information and actual signal components.

5. CONCLUSION

Drawing from Rényi complexity measures and mutual information theory, health condition of the hanger bearing and gearbox are quantitatively distinguished by the proposed mutual information technique. New normalization of cross time-frequency distribution techniques were proposed along with some clarifications of previously used methods. This allowed for a mutual information method to be derived that is more in line with the results seen from a self information or entropy measure. Owing to the proposed use time-frequency based information measure technique, new baseline comparisons were drawn for bearing mutual information values. Scalar valued condition indicators are proposed using mean values, self-information, and mutual information. We have compared the utility of self-information and mutual information in both bearing and gearbox vibration analysis for condition indicators. It has been shown that diagnostics obtainable from time-frequency representations can be indicated by these proposed CI self and mutual information measures.

We have extended methods of self and mutual information to gearbox vibrations signatures. Moreover, we have been able to analyze the relationship of torque and speed to the results of our mutual indicator methods. The effects of speed and torque may be minimal to the mutual information method, and given knowledge of these factors from a tachometer input and torque transducer, an algorithm could be derived to compensate for these effects. The mutual information method could be used as an advanced condition indicator to show increased amounts of common time-frequency signatures as reported by two separate sensors indifferent of torque. This could be useful in applications such as active vibration cancellation and a fused sensor fault diagnostics.

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**Abdel E. Bayoumi** is Director of the USC Biomedical Engineering Program and Professor of Mechanical Engineering at the University of South Carolina-Columbia. He chaired the Department of Mechanical Engineering from 1998 through 2006. Before joining USC, he was a Professor and Director of the Manufacturing Program at North Carolina State University-Raleigh, North Carolina (1996–1998); Assistant, Associate, Professor, and Distinguished Boeing Manufacturing Professor at Washington State University (1983-1996); a Project Manager at Hewlett-Packard Company-Corvallis, Oregon (1993–1995 – on professional leave from WSU); and a visiting scholar for a year at the American University in Cairo (AUC) - Egypt (1991-1992 – a sabbatical leave from WSU). During his tenure at the University of South Carolina, North Carolina State University, Washington State University, the American University in Cairo, and Hewlett-Packard Company, Dr. Bayoumi has been actively involved in developing strong research and educational programs. His current areas of interest can be grouped into three categories, (1) Study of Condition-Based Maintenance (CBM) of military aircraft in which diagnosis, prognosis and health monitoring systems are effectively utilized using informatics and sensing technologies, (2) Micro-Electro Mechanical Systems (MEMS) and Mechatronics in which a MEMS device is designed, fabricated and used to sense and control mechanical or biological systems, and (3) Design and applications of efficient energy resources and system.