Problem of the Month

September 2017
Solution

Largest of Equals

Find the area of the largest equilateral triangle that fits in a square of side length 1.

Solution: The area is $2\sqrt{3}-3$.

Proof. The area is $2\sqrt{3}-3$ and can be obtained by putting one vertex at the bottom left corner of the square, and the other two vertices on the top and right sides at a distance $1-d$ from the top right corner, where $d=2-\sqrt{3}$.

To derive the value of d we simply solve the equation $1+d^2 = 2(1-d)^2$, choosing the solution which is less than 1. The two sides of the equation are the squares of two of the sides of the triangle formed by the lower left corner of the square and point on the top and rights sides of the square a distance $1-d$ from the top right corner. Using the Pythagorean Theorem we get the left hand side of the equation by going across from the bottom left corner to the bottom right corner a distance of 1, then up a distance of d. The right hand side of the equation is found using the distance squared between the two points on the top and right-hand edges. As the triangle is to be equilateral, these two measurements are to be the same.

Not that if an equilateral triangle with vertices on the edges of a square does not have even one of those vertices at a corner of the square, then a slight rotation about the center of the triangle will move all vertices to the interior of the square. The equilateral triangle can then be enlarged while still remaining in the square. In this way we see that the largest equilateral triangle must have one of its vertices on a corner of the square. ■

Congratulations to this month's winner!

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