JANURAY/FEBRUARY SOLUTIONS

SUM OF THE PRODUCT OF SUBSETS

Let $A=\{2, 3, 5, 7, 11, 13\}$, and let $A_1, A_2, ..., A_{63}$, be all of the nonempty subsets of *A*. Let $f(M)$ denote the product of all the elements in *M*. Compute $f(A_1)+f(A_2)+...+f(A_{63})$

SOLUTION

The key to this problem is the relationship

$$f(\{a_1,a_2,...,a_n\})=a_n f(\{a_1,a_2,...,a_{n-1}\}).$$

Order the elements of *A* as follows: $a_1=2, a_2=3,..., a_6=13$. Let S_n denote the sum of all terms $f(A_i)$ in which A_i is a subset that includes only members of the first *n* elements of *A*. Grouping the sets including only the first *n* elements by whether they include the n^{th} element gives us

$$S_n=a_nS_{n-1}+S_{n-1}+a_n=(a_n+1)S_{n-1}+a_n$$

Thus we have

$$S_1=2$$
$$S_2=(3+1)\cdot 2+3=11$$
$$S_3=(5+1)\cdot 11+5=71$$
$$S_4=(7+1)\cdot 71+7=575$$
$$S_5=(11+1)\cdot 575+11=6{,}911$$
$$S_6=(13+1)\cdot 6911+13=96{,}767$$

A more direct solution is to observe that the sum of the products can be expressed in terms of the original elements as *A* as follows:

$$(a_1+1)\cdot(a_2+1)\cdot ...\cdot(a_n+1)-1 = 96{,}767.$$

CORRECT SOLUTIONS

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