

## FEBRUARY/MARCH 2012 SOLUTIONS

### SUM OF LOGS

For any real number  $x$ , let  $\lfloor x \rfloor$  denote the greatest integer less than or equal to  $x$ . Without relying on technology, compute the sum

$$\lfloor \log_3 1 \rfloor + \lfloor \log_3 2 \rfloor + \dots + \lfloor \log_3 1,000 \rfloor.$$

### SOLUTION

Observe for  $1 \leq n < 3$ , that  $\lfloor \log_3 n \rfloor = 0$ . Similarly for  $3 \leq n < 3^2 = 9$ , we have  $\lfloor \log_3 n \rfloor = 1$ , and for  $3^2 \leq n < 3^3$ , the quantity  $\lfloor \log_3 n \rfloor = 2$ . In general for  $3^k \leq n < 3^{k+1}$ ,  $\lfloor \log_3 n \rfloor = k$ . Note that  $3^6 < 1000 < 3^7$ , so the sum is equal to

$$\begin{aligned} & 1 \cdot (9 - 3) + 2 \cdot (27 - 9) \\ & + 3 \cdot (81 - 27) + 4 \cdot (243 - 81) \\ & + 5 \cdot (729 - 81) + 6 \cdot (1000 - 729) + 1 \\ & = \mathbf{4914} \end{aligned}$$

### CORRECT SOLUTIONS

- (1) Daniel Grier
- (2) Nolan Miller
- (3) Stefan Singer
- (4) Daniel Wallis
- (5) Elizabeth Minten
- (6) David Hughey
- (7) Robert Moyer
- (8) Ryan Benitez
- (9) Runyu Bi