

OCTOBER/NOVEMBER SOLUTIONS

DIRECTION OF TRAVELING

The city of Orange is arranged in a 10 block grid. There is a park that covers 2 blocks of the city, and it is located on the 4th and 5th blocks from the west side of the city and 3rd block from the south side. There are no roads through the park. A car starts at the southwest corner of the city and can go either north or east at each intersection. How many ways are there for the car to reach the northeast corner?

SOLUTION

Let N denote the car traveling north at a specific intersection and E denote the car traveling east. In order for the car to reach the destination, we need to travel north 10 times and east 10 times. Thus we can describe the path as a sequence of 10 N's and 10 E's. Therefore there are $C(20,10) = 184,756$ ways, assuming that there is a road through the park. Now, we need to calculate the number of paths that use the park road. In order to use the park road the car must be between the 4th and 5th blocks from the west side of the city and between the 2nd and 3rd blocks from the south side of the city, i.e. the car must move 2 blocks north and 4 blocks east of the starting position. There are $C(6,2) = 15$ ways to do this. After using the road through the park, the car must be between the 4th and 5th blocks from the west side of the city and between the 3rd and 4th blocks from the south side of the city. Thus there are $C(13,7) = 1,716$ ways for the car to reach the destination after leaving the park. Thus by the principle of counting, there are $15 * 1,716 = 25,740$ paths from start to finish using the road through the park. Hence there are $184,756 - 25,740 = 159,016$ ways to drive from start to finish that do not go through the park.

CORRECT SOLUTIONS

- (1) George Helman
- (2) Daniel Grier