

## NOVEMBER/DECEMBER SOLUTIONS

### REMAINDER OF A POLYNOMIAL

Let  $p(x) = x^{2011} + x^{1783} - 3x^{1707} + 2x^{341} + 3x^2 - 3$ . Without using a calculator or computer, find the remainder when  $p(x)$  is divided by  $x^3 - x$ . Show all work on this problem.

### SOLUTION

By the division algorithm we can write

$$p(x) = (x^3 - x)q(x) + r(x)$$

where  $r(x)$  is the remainder term. Since  $x^3 - x$  has degree 3, the remainder  $r(x)$  must have a degree of at most two. Thus there exists integers  $a$ ,  $b$ , and  $c$  such that  $r(x) = ax^2 + bx + c$ . Now, in order to determine  $a$ ,  $b$ , and  $c$ , we let  $x$  be  $-1$ ,  $0$ , and  $1$  to obtain the following system of equations

$$-1 = p(-1) = 0 \cdot q(-1) + r(-1) = a - b + c$$

$$-3 = p(0) = 0 \cdot q(0) + r(0) = c$$

$$1 = p(1) = 0 \cdot q(1) + r(1) = a + b + c$$

By solving the linear system we find that  $a = 3$ ,  $b = 1$ , and  $c = -3$ .

Therefore the remainder is  $r(x) = 3x^2 + x - 3$

### CORRECT SOLUTIONS

- (1) Cole Franks
- (2) Daniel Grier
- (3) Reid Harris