SOLUTION (Continued Radicals)

By squaring both sides, we get

$$C^2 = m + \sqrt{m + \sqrt{m + \sqrt{m + \ldots}}} = m + C,$$

or $C^2 - C - m = 0$, with positive solution

$$C = \frac{1 + \sqrt{1 + 4m}}{2}.$$

In order to have C a positive integer, we must have $1 + 4m = k^2$, where k is an odd integer. This implies that

$$m = \frac{k^2 - 1}{4} = \left(\frac{k-1}{2}\right)\left(\frac{k+1}{2}\right).$$

This means that m must be the product of two consecutive positive integers.

SOLUTION (Non-square Factors)

Since $36{,}000{,}000 = 2^8 3^2 5^6$, every positive factor must be of the form $2^p 3^q 5^r$, where $p \in \{0, 1, 2, \ldots, 8\}$, $q \in \{0, 1, 2\}$, and $r \in \{0, 1, 2, \ldots, 6\}$. Therefore, there are $9 \times 3 \times 7 = 189$ factors. The factors which are perfect squares have the form $(2^2)^j (3^2)^k (5^2)^l$, where $j \in \{0, 1, 2, 3, 4\}$, $k \in \{0, 1\}$, and $l \in \{0, 1, 2, 3\}$, so there are $5 \times 2 \times 4 = 40$ of these. Thus, the number of factors that are *not* perfect squares is $189 - 40 = 149$.