

MARCH/APRIL SOLUTIONS

PROBLEM

Calculate the derivative with respect to x of the function

$$y = x^{x^x}.$$

SOLUTION

$$\begin{aligned}y &= x^{x^x} \\ \ln y &= \ln x^{x^x} = x^x \ln x \\ \ln \ln y &= \ln(x^x \ln x) = \ln x^x + \ln \ln x \\ &= x \ln x + \ln \ln x\end{aligned}$$

Differentiating both sides,

$$\begin{aligned}\frac{1}{\ln y} * \frac{1}{y} * y' &= x * \frac{1}{x} + \ln x + \frac{1}{\ln x} * \frac{1}{x} \\ &= 1 + \ln x + \frac{1}{x \ln x} \\ y' &= y \ln y \left(1 + \ln x + \frac{1}{x \ln x} \right) \\ &= x^{x^x} x^x \ln x \left(1 + \ln x + \frac{1}{x \ln x} \right) \\ &= x^{x^x} (x^x \ln x + x^x (\ln x)^2 + x^{x-1}).\end{aligned}$$

CORRECT SOLUTIONS

- (1) Andrew Shore
- (2) John Holt
- (3) Holly Watson (Answer Only)
- (4) Jim Manning