Sergey Fomin and Michael Shapiro proved that the totally nonnegative real part of the unipotent radical of a Borel in a semisimple, simply connected algebraic group has a cell decomposition with Bruhat order as its poset of closure relations, and they conjectured that (after deconing) this was a regular CW complex homeomorphic to a closed ball. Much of the interest in these spaces comes from their interpretation as images of maps related to Lusztig's theory of canonical bases.

I will briefly discuss my proof of this conjecture, then turn to new joint work with Jim Davis and Ezra Miller regarding the structure of the fibers of these maps. This will include telling much of the back-story leading up to this work as well as providing motivation and background in this area along the way.