A grand challenge with great opportunities is to develop a coherent framework that enables blending conservation laws, physical principles, and/or phenomenological behaviours expressed by differential equations with the vast data sets available in many fields of engineering, science, and technology. At the intersection of probabilistic machine learning, deep learning, and scientific computations, this work is pursuing the overall vision to establish promising new directions for harnessing the long-standing developments of classical methods in applied mathematics and mathematical physics to design learning machines with the ability to operate in complex domains without requiring large quantities of data. To materialize this vision, this work is exploring two complementary directions: (1) designing data-efficient learning machines capable of leveraging the underlying laws of physics, expressed by time dependent and non-linear differential equations, to extract patterns from high-dimensional data generated from experiments, and (2) designing novel numerical algorithms that can seamlessly blend equations and noisy multi-fidelity data, infer latent quantities of interest (e.g., the solution to a differential equation), and naturally quantify uncertainty in computations. The latter is aligned in spirit with the emerging field of probabilistic numerics.