The Application and Promise of Hierarchical Linear Modeling (HLM) in Studying First-Year Student Programs

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Southern Illinois University Carbondale

- Large doctoral research public university located in the southern tip of Illinois
- Six hours from Chicago
- Rural community
- Large number of students from the Chicago land area

Enrollment

- On–Campus Enrollment at the time of study
  - 19,124
- On-Campus Residence Hall Enrollment at the time of study
  - 4,314 students

Primary Purpose and Applications for Hierarchical Linear Modeling (HLM)

- HLM allows us to assess and model the variable effects of context or environment
- Example Housing and FYE Applications
  - Students nested within
    - Classrooms (e.g., freshman seminars)
    - Programs (e.g., LLCs or Peer Mentoring)
    - Residence halls and floors
    - Universities (cross-institutional research)

Additional Applications for HLM

- Item Analysis
  - Items nested within respondents
- Growth Modeling or Longitudinal Research
  - Observations over time nested within students
- Cross-Classification
  - Students nested within more than one group (e.g., floors and classrooms, or different living environments across time)
- Meta-Analysis
  - Coefficients nested within studies

Failing to Capture the Social Ecology of First-Year Experience Programs

- Participation in FYE programs typically takes place within a group context AND this context often influences individual outcomes
  - In fact, Living-Learning Communities (LLCs) are designed to capitalize on the resources and dynamics of group membership to yield desired outcomes (e.g., GPA and persistence)
- Yet, FYE and Housing evaluation efforts rarely use HLM to model the influence of these "context effects"

Literature Review

- Located Housing and First-Year Experience (FYE) articles that used HLM in their analysis
  - 4 major databases were searched
    - EBSCO, ERIC JSTOR and MUSE
  - Keywords for <u>HLM</u>, <u>Housing</u> and <u>First-Year Experience</u> programs were cross-referenced in each database
- Results
  - Just 5 articles* were found

* Please contact presenters for references

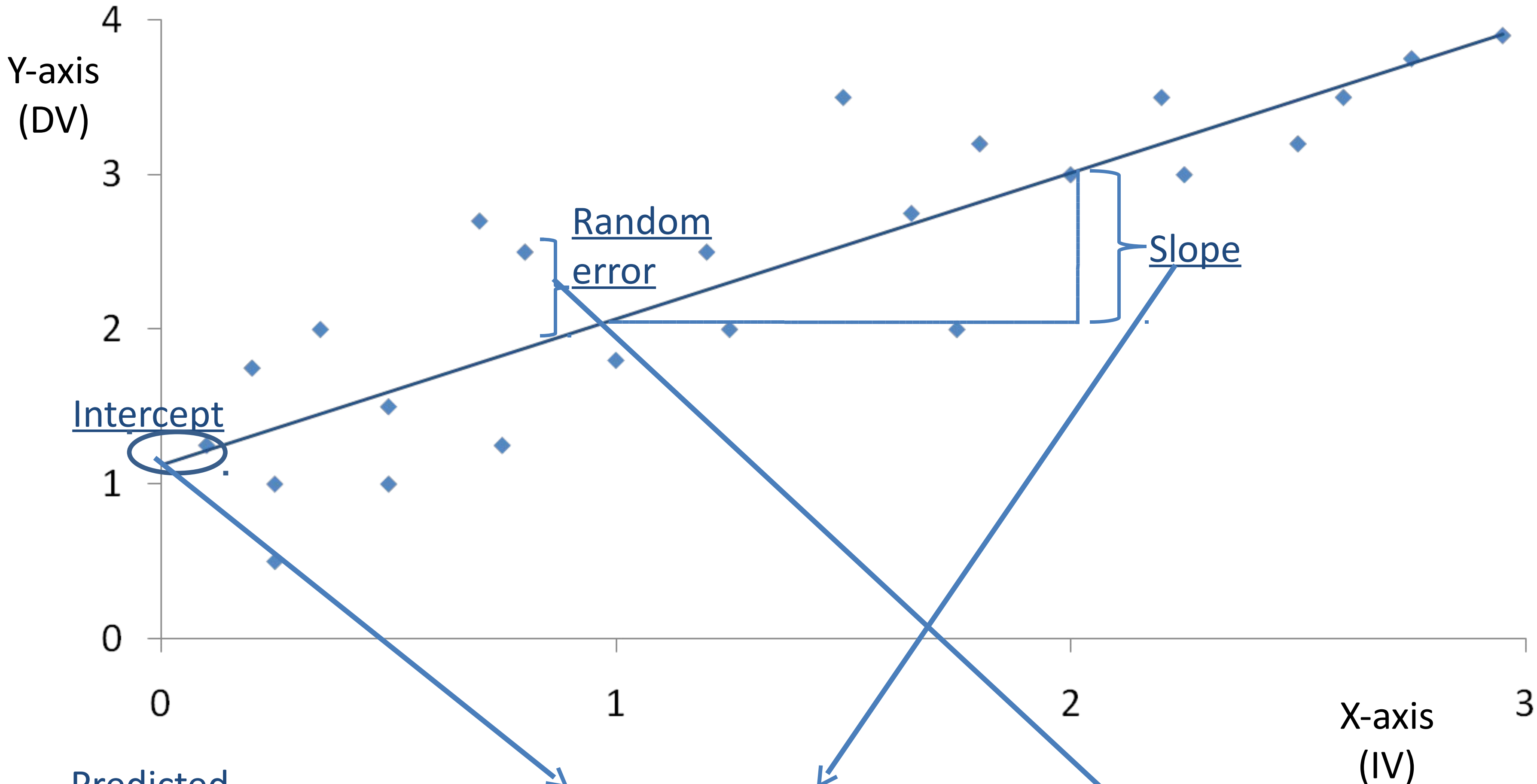
Traditional Methods of Modeling Context (or Group-Level) Effects

- Disaggregation of Group Characteristics
  - Group characteristics are assigned to everyone in a group
  - Violates assumption of independence
- Aggregation of Individual Characteristics
  - Mean individual characteristics assigned to group
  - Loss of sample size, within-group variability and power
- Consequence
  - Biased estimates of effect → inaccurate/misleading results

Hierarchical Linear Modeling (HLM)

- HLM allows us to obtain unbiased estimates of effect for group context variables
- *Hierarchical*
  - Indicates that Level 1 (or student-level) coefficients become outcomes at Level 2 (group-level)

Regression Refresher



Predicted outcome for Student "i" →

$$Y_i = B_o + B_1(X_i) + r_i$$

Linear Model Terminology

- Where,
  - Y_{ij} = Outcome for person i in group j
  - β_{0j} = Intercept for group j
    - Value of Y when X = mean of group j
    - If X is centered around the grand mean ($X_{ij} - \overline{X}_{..}$), then the intercept equals the value of Y for a person with an X equal to the average X across all groups
  - β_{1j} = Slope for group j
    - Change in Y associated with a 1 unit change in X
  - $X_{ij} - \overline{X}_{.j}$ = group-mean centered value of Level 1 variable for person i in group j
  - r_{ij} = random error term (predicted Y_{ij} – observed Y_{ij}) for person i in group j
    - $r_{ij} \sim N(0,\sigma^2)$, or
    - r_{ij} is assumed normally distributed with a mean of "0" and a constant variance equal to sigma-squared.

Regression with Multiple Groups

- Two Group Case

$$Y_{i1} = B_{01} + B_{11}(X_{i1}) + r_{i1}$$

$$Y_{i2} = B_{02} + B_{12}(X_{i2}) + r_{i2}$$

- J Group Case

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \overline{X}_{.j}) + r_{ij}$$

Two-Level Model

- Level 1 Equation

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \overline{X}_{.j}) + r_{ij}$$

- Level 2 Equations

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j}$$

Mixed Two-Level Model

Grand Mean

Main Effects of W_j and X_{ij}

$$Y_{ij} = \gamma_{00} + \gamma_{01}(W_j) + \gamma_{10}(X_{ij} - \overline{X}_{.j})$$

Interaction Effect of W_j and X_{ij}

$$+ \gamma_{11} W_j (Xi_j - \overline{X}_{.j})$$

Random Error Terms for Intercept, Slope and Student

$$+ \mu_{0j} + \mu_{1j}(X_{ij} - \overline{X}_{.j}) + r_{ij}$$

Two-Level Model Terminology

- Generally,
  - γ_{00}= Grand Intercept (mean of Y across all groups)
  - γ_{01}= avg. difference between Grand Intercept and Intercept for group j given W_j
  - γ_{10}= Grand slope (slope across all groups)
  - γ_{11}= avg. difference between Grand Slope and slope for group j given W_j and X_{ij}
  - μ_{0j} = random deviation of group means about the grand intercept
  - μ_{1j} = random deviation of group slopes about the grand slope
  - W_j = Level 2 predictor for group j

Variance-Covariance Components

- $Var(r_{ij}) = \sigma^2$
  - Within-group variability
- $Var(\mu_{0j}) = \tau_{00}$
  - Variability about grand mean
- $Var(\mu_{1j}) = \tau_{11}$
  - Variability about grand slope
- $Cov(\mu_{0j}, \mu_{1j}) = \tau_{01}$
  - Covariance of slopes and intercepts

EXAMPLE ANALYSIS

Background on Housing Hierarchy

- University Housing Halls/Floors
  - Brush Towers
    - 32 Floors
    - 2 Halls
  - Thompson Point
    - 33 Floors
    - 11 Halls
  - University Park
    - 56 Floors
    - 11 Halls
- Totals
  - 121 Floors
  - 24 Halls

Living-Learning Community (LLC) Program at SIUC in 2005

- LLC Program Components
  - Academic/Special Emphasis Floors (AEFs)
    - First Implemented in 1996
    - 12 Academic/Special Emphasis Floors
  - Freshman Interest Groups (FIGs)
    - First implemented in 2001
    - 17 FIGs offered
    - Some FIGs were nested on AEFs

Data Collection

- All example data were collected via university records
- Sampling
  - LLC Students (n = 421)
    - All FIG students (n = 223)
    - Random sample of AEF students (n = 147)
    - All FIG students nested on AEFs (n = 51)
  - Random sample of Comparison students (n = 237)

2005 Cohort
Sample Demographics

Group	N	Percent Female	Percent White	Percent African American	Mean ACT
LLC	421	38%	64%	25%	22.44
Comparison	237	46%	57%	36%	21.21
Total	658	41%	62%	31%	22.00

Hierarchical 2-Level Dataset

- Level 1
  - 657 Students
- Level 2
  - 93 Floors
    - 1 to 31 students (mean = 7) populated each floor
    - Low n-sizes per floor are not ideal, but HLM makes it possible to estimate coefficients with some accuracy via Bayes estimation.

Variables Included in Example Analysis

- Outcome (Y)
  - First-Semester GPA (Fall 2005)
- Student-Level Variables (X's)
  - Student ACT score
  - Student LLC Program Participation (LLC student = 1, Other = 0)
- Floor-Level Variables (W's)
  - MEAN_ACT (average floor ACT score)
  - LLC Participation Rate (Percent LLC students on floor)

Research Questions Addressed in Example Analysis

- How much do residence hall floors vary in terms of first-semester GPA?
- Do floors with high MEAN ACT scores also have high first-semester GPAs?
- Does the strength of the relationship between the student-level variables (e.g., student ACT) and GPA vary across floors?
- Are ACT effects greater at the student- or floor-level?
- Does participation in an LLC (and participation rate per floor) influence first-semester GPA after controlling for student- and floor-level ACT?
- Are there any student-by-environment interactions?

Model Building

- HLM involves five model building steps:
  1. One-Way ANOVA with random effects
  2. Means-as-Outcomes
  3. One-Way ANCOVA with random effects
  4. Random Intercepts-and-Slopes
  5. Intercepts-and-Slopes-as-Outcomes

One-Way ANOVA with Random Effects

LEVEL 1 MODEL

$$F05GPA_{ij} = \beta_{0j} + r_{ij}$$

Level-1 Slope is set equal to 0.

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

MIXED MODEL

$$F05GPA_{ij} = \gamma_{00} + u_{0j} + r_{ij}$$

ANOVA Results and Auxiliary Statistics

- Point Estimate for Grand Mean
  - γ_{00} = 2.59***
- Variance Components
  - σ^2 = .93
  - τ_{00} = .05*
    - Because τ_{00} is significant, HLM is appropriate
- Auxiliary Statistics
  - Plausible Range of Floor Means
    - 95%CI = γ_{00} +- 1.96$(\tau_{00})^{½}$ = 2.13 to 3.05
  - Intraclass Correlation Coefficient (ICC)
    - ICC = .06 (6% of the var. in first-semester GPA is between floors)
  - Reliability (of sample means)
    - λ =.24

Means-as-Outcomes

LEVEL 1 MODEL

$$F05GPA_{ij} = \beta_{0j} + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(MEAN\_ACT_j) + u_{0j}$$

Level-2 Predictor Added to Intercept Model

MIXED MODEL

$$F05GPA_{ij} = \gamma_{00} + \gamma_{01} * MEAN\_ACT_j + u_{0j} + r_{ij}$$

Means-as-Outcomes
Results and Auxiliary Statistics

- Fixed Coefficients
  - γ_{01} = 0.92*** (effect of floor-level ACT)
- Variance Components
  - σ^2 = .93
  - τ_{00} = .04 (ns, p = .11)
- Auxiliary Statistics
  - Proportion Reduction in Variance (PRV)
    - PRV = .28 (MEAN ACT accounted for 28% of between-group var.)
  - Conditional ICC
    - ICC = .04 (Remaining unexplained variance between floors = 4%)
  - Conditional Reliability
    - λ = .20 (reliability with which we can discriminate among floors with identical MEAN ACT values)

One-Way ANCOVA with Random Effects

LEVEL 1 MODEL

$$F05GPA_{ij} = \beta_{0j} + \beta_{1j}(ACT_{ij} - \overline{ACT}_{\cdot j}) + r_{ij}$$

Level-1 Covariate Added to Model

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

MIXED MODEL

$$F05GPA_{ij} = \gamma_{00} + \gamma_{10} * (ACT_{ij} - \overline{ACT}_{\cdot j}) + u_{0j} + r_{ij}$$

ANCOVA
Results and Auxiliary Statistics

- Fixed Coefficients
  - γ_{10} = 0.07*** (effect of student ACT)
- Variance Components
  - σ^2 = .88
- Auxiliary Statistics
  - PRV due to Student SES
    - PRV = .05
      - Student ACT accounted for 5% of the within-group variance
      - MEAN ACT accounted for 28% of the between-group variance
        - » Thus, ACT seems to have more of an influence on first-semester GPA at the group (or floor) level than at the individual-level

Random Coefficients

LEVEL 1 MODEL

$$F05GPA_{ij} = \beta_{0j} + \beta_{1j}(ACT_{ij} - \overline{ACT}_{\cdot j}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

Both Intercepts and Slopes are Set to Randomly Varying Across Level-2 Units

MIXED MODEL

$$F05GPA_{ij} = \gamma_{00} + \gamma_{10} * (ACT_{ij} - \overline{ACT}_{\cdot j}) + u_{0j} + u_{1j} * (ACT_{ij} - \overline{ACT}_{\cdot j}) + r_{ij}$$

Random Coefficients Results

- Variance-Covariance Components
  - σ^2 = .88
  - τ_{00} = .06**
  - τ_{11} = .00 (ns, p = .40)
    - Because τ_{11} is non-sig., slopes are constant across floors, and term can be dropped
    - Dropping τ_{11} also increases efficiency because μ_{1j}, τ_{11} and τ_{01} don't have to be estimated.
  - τ_{01} = .001

Random Coefficients Auxiliary Statistics

- Auxiliary Statistics
  - Reliability of Intercepts and Slopes
    - $\lambda(\beta_0) = .31$
      - Reliability with which we can discriminate among floor means after student SES has been taken into account
    - $\lambda(\beta_1) = .00$
      - Reliability with which we can discriminate among the floor slopes; in this case, the grand slope adequately describes the slope for each floor.
  - Correlation Between Floor Intercepts and Slopes
    - $\rho = .66$
      - Floors with high MEAN ACT scores also have high mean first-semester GPAs

Intercepts-and-Slopes-as-Outcomes (ISO)

Added student-level participation in LLC program to Level 1 model; set slope to non-randomly vary across floors

LEVEL 1 MODEL

$$F05GPA_{ij} = \beta_{0j} + \beta_{1j}(LLC_{ij}) + \beta_{2j}(ACT_{ij} - \overline{ACT}_{\cdot j}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(LLC_{j}) + \gamma_{02}(MEAN\_ACT_{j}) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(LLC_{j}) + \gamma_{12}(MEAN\_ACT_{j})$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}(LLC_{j}) + \gamma_{22}(MEAN\_ACT_{j})$$

I-S-O Mixed Model

MIXED MODEL

$$F05GPA_{ij} = \gamma_{00} + \gamma_{01}(LLC_j) + \gamma_{02}(MEANACT_j)$$
$$+ \gamma_{10}(LLC_{ij}) + \gamma_{11}(LLC_j)(LLC_{ij}) + \gamma_{12}(MEANACT_j)(LLC_{ij})$$
$$+ \gamma_{20}(ACT_{ij} - \overline{ACT}_{.j}) + \gamma_{21}(LLC_j)(ACT_{ij} - \overline{ACT}_{.j})$$
$$+ \gamma_{22}(MEANACT_j)(ACT_{ij} - \overline{ACT}_{.j})$$
$$+ \mu_{0j} + r_{ij}$$

Interpretation of Mixed I-S-O Model

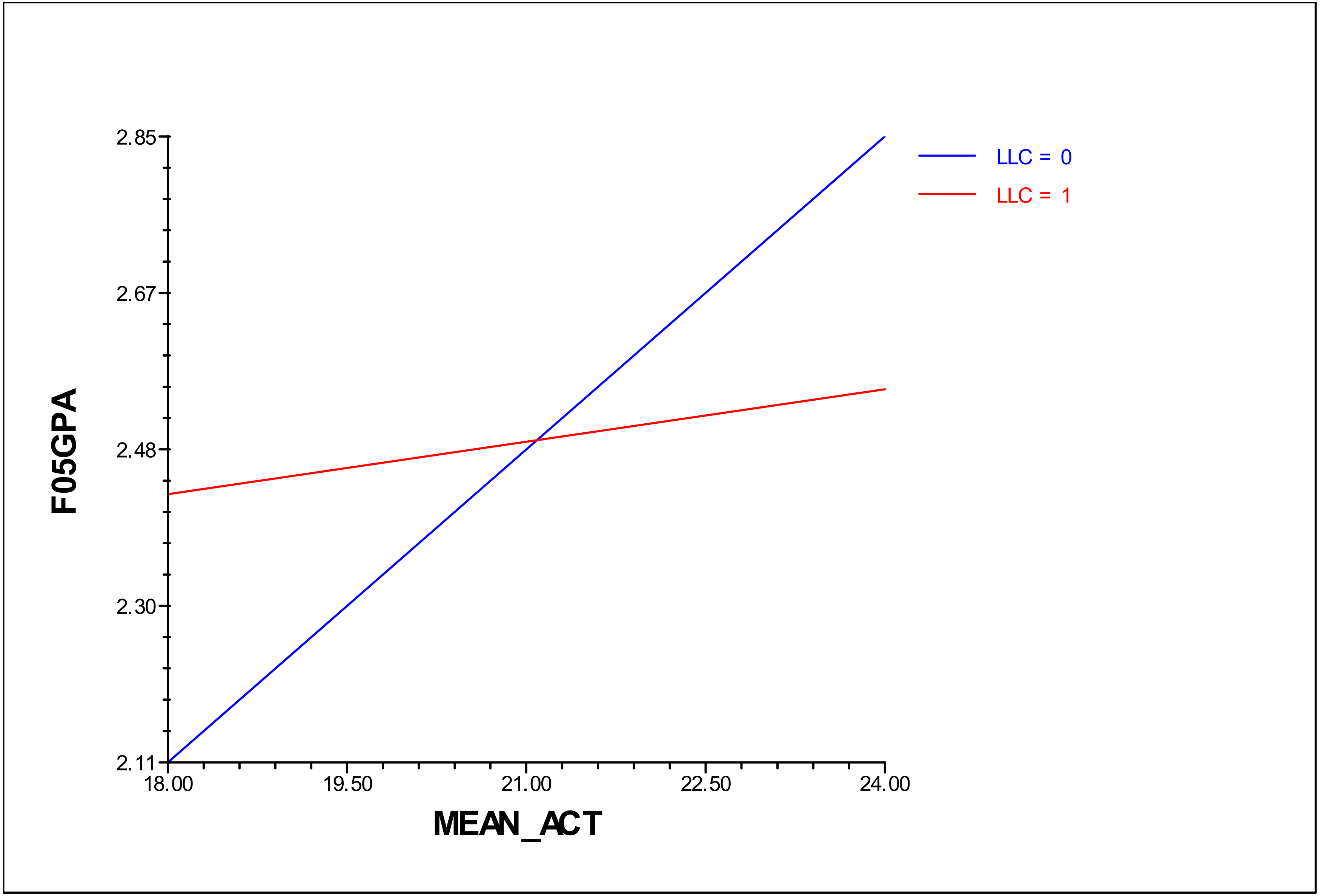
- First-Semester GPA =
  - Grand Mean
  - 4 Main Effects
    - Percentage of Students on Floor Participating in an LLC
    - Floor's Mean ACT score
    - Student-Level Participation in a LLC
    - Student's ACT score
  - 4 Interaction Terms
    - Floor LLC by Student LLC
    - Floor ACT by Student LLC
    - Floor LLC by Student ACT
    - Floor ACT by Student ACT
  - 2 Random Error Components
    - Residual deviation about grand mean
    - Residual deviation about floor mean

I-S-O Results

Fixed Effects	Coefficient	SE	Sig.
Grand Intercept, γ_{00}	-0.18	0.58	ns
Main Effects			
LLC Floor-Level Participation Rate, γ_{01}	0.15	0.22	ns
Floor MEAN ACT, γ_{02}	0.12	0.03	***
Student LLC Participation, γ_{10}	2.03	0.83	*
Student-Level ACT (Grand Slope), γ_{20}	-.07	.14	ns
Interaction Effects			
Floor LLC Rate by Student LLC, γ_{11}	0.29	0.32	ns
Floor MEAN ACT by Student LLC, γ_{12}	-0.10	0.04	**
Floor LLC Rate by Student ACT, γ_{21}	0.07	0.03	*
Floor MEAN ACT by Student ACT, γ_{22}	0.00	0.01	ns

*** $p < .001$ ** $p < .01$ * $p < .05$ ^ $p < .10$

Floor MEAN ACT by Student LLC



Caveats of Using HLM

- Large Sample Sizes
  - Sample sizes of j by n can quickly get out of hand (and expensive)
  - Large n- and j-sizes not required, but reliability of estimates increase as n and effect size increase
    - No "magic" number, but central limit theorem suggests that (multivariate) normality can be achieved with around 30 per group, with 30 groups
- Advanced Statistical Procedure
  - Use of HLM requires:
    - A solid background in multivariate statistics
    - Time to learn the statistical language
  - SSI offers seminars, but statistical language should be familiar before attending
- HLM is only a Statistical Technique
  - HLM's efficacy is limited by quality of research design and data collection procedures

Resources and Costs

- Scientific Software International (SSI) - www.ssicentral.com
- HLM 6.06 software (single license = $425)
  - Free student version available
- Software User's Manual ($35)
  - Raudenbush, S., Bryk, A., Cheong, Y. F., & Congdon, R. (2004). *HLM 6: Hierarchical Linear and Nonlinear Modeling.* Scientific Software International: Lincolnwood, IL.
- Textbook ($85)
  - Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical Linear Models: Applications and Data Analysis Methods*. Sage Publications: Thousand Oaks, CA.

Presentation Reference

- Briggs, C. S., Lorentz, K., & Davis, E. (2009). *The application and promise of hierarchical linear modeling in studying first-year student programs*. Presented at the annual conference on The First-Year Experience, Orlando, FL.

Further Information

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