Sample January 2018 Applied Solutions

(a) No, the distribution has light tails.

(b) No, the √ or log will help "Fan-shaped" heterogeneity and some types of non-linearity. They don't particularly help light tails.

c) Ho: μ=24 vs. Ha: μ < 24 or μ ≥ 24

d) \[ T = \frac{21.20 - 24}{3.81 / \sqrt{14}} = -2.750 \]

\[ \alpha = .05 \]

Reject Ho.

We conclude the average length is too small.

\[ 21.20 ± t_{.05,13} \frac{3.81}{\sqrt{14}} = (18.994, 23.406) \]

Better is the upper confidence bound

\[ (-\infty, 21.20 + t_{.05,13} \frac{3.81}{\sqrt{14}}) = (-\infty, 23.008) \]

(e) 21.20 ± 6.025,13 \[ \frac{3.81}{\sqrt{14}} \]

\[ \text{Better is the upper confidence bound} \]

\[ \text{(-∞, 21.20 + t_{0.05,13} \frac{3.81}{\sqrt{14}}) = (-∞, 23.008)} \]

(f) All three tests work on the mean for symmetric distributions. In general the Wilcoxon test is probably better - much better than sign for light tails, much better than t for heavy, and never much worse than t. For light tails it is almost the same as t though. Here its one-sided p-value would be \[ \frac{10785}{203925} \leq \alpha = .05 \text{ and we still reject Ho.} \]
2a)  \( H_0: \pi_S = \pi_L \quad \pi = \% \) who would give thumbs up watching that video

\[ \pi_S = \frac{84}{256} \quad \pi_L = \frac{59}{244} \quad \bar{\pi} = \frac{84 + 59}{256 + 244} \]

\[ \approx 0.3281 \approx 0.2418 \approx 0.286 \]

\[ Z = \frac{(0.3281 - 0.2418) - 0}{\sqrt{0.286(1-0.286) + 0.286(1-0.286)}} \approx 2.1346 \]

Could also use \( \chi^2 \) test for homogeneity

Using RR 2.1346 > 1.645.

\( p \)-value \( \approx 1.9834 \approx 0.0162 < 0.05 \)

Reject \( \text{Ho} \) and conclude we have sufficient evidence that the shorter video leads to more thumbs up.

b) Random allocation was used and the expected counts of 256 (.286), 256(1-.286), 244(.286), and 244(1-.286) are all significantly larger than 5.

c) In the test the null hypothesis is \( \pi_S = \pi_L \), and we conduct tests so that only a single \( \pi \) value should go in to \( \pi_S \) and \( \pi_L \) in the denominator. In the CI, that assumption isn't there and so we can't assume \( \pi_S = \pi_L \).

d) The formula with \( \pi_S \) and \( \pi_L \) is already approximate because it uses the CLT and doesn't use a continuity correction, and assumes the \( \pi \) equal the \( \bar{\pi} \). The "fake data" in the Agresti - Caffo Correction give much closer to actually being 95.4% confidence than not using it.
3a) The participants were self-selected (volunteers). This makes it hard to generalize.
Unequal sample sizes would make it not robust to heterogeneity.

b) Variances might differ some (which is bad due to differing sample sizes).
Symmetry (mean 0) seems to hold.
This is not a great graph for checking assumptions.

c) With a p-value of 0.06 from the ANOVA table, at $\alpha = 0.05$ we would fail to reject H0. We do not have significant evidence of improvement.
($\alpha = 0.10$ would give a different conclusion).

d) Tukey-Kramer adjusts for unequal sample sizes and is generally more powerful than Bonferroni for similar number of comparisons and much more powerful than Scheffe.

e) The p-values are: Elem vs. Int 0.1122
Elem vs. Nov 0.7738
Significant difference $\rightarrow$ Novice vs. Elem 0.0477

Tukey-Kramer demonstrates its power by being the only one to find this difference.
4a) Factorial because each variety-fertilizer combination occurs.

Balanced because each combination that occurs happens 3 times (the same number)

With replications because each combination that occurs happens more than once.

Mixed effect because there are three particular varieties (fixed effect), but the fertilizers were of interest randomly selected from the possible ones (random effect)

b) \( Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \)

\( \tau \) - response
\( \beta \) - overall effect of variety i
\( \epsilon \) - error
\( \tau\beta \) - interaction of variety i and fertilizer j
\( \epsilon \) - error

This is the usual F test

with \( F = 16.97 \) and \( df \) 2, 12

from either the Type I or Type III tests.

\( H_0 : \tau_1 = \tau_2 = \tau_3 \)

vs. \( H_a : \) at least one \( \tau \) differs

d) Need to adjust for this one using the correct variety and error lines of the set-up where both are random.

\( F = \frac{MS_{\text{variety}}}{MS_{\text{variety} \times \text{fertilizer}}} = \frac{1123.1950}{432.7572} = 25.91 \)

with \( df = 2, 2. \)

\( H_0 : \sigma^2 = 0 \) vs. \( H_a : \sigma^2 \neq 0 \)
5a) \( p \)-value is < 0.001. We reject the null hypothesis that log weight does not linearly predict log brain. (reject \( H_0: B_{\text{logw}} = 0 \) in favor of \( H_1: B_{\text{logw}} \neq 0 \)).

b) \( R^2 = 0.923912 \)

The linear regression on log weight explains around 92.4% of the variability in log brain.

c) \( \sqrt{MSE} = 0.524855 \)

The standard deviation of the errors (distance of the log brain from the true regression line) is 0.525, approximately.

d) The Type I test tests that \( H_0: B_{\text{litter}} = 0 \) given log weight is included in the model vs. \( H_1: B_{\text{litter}} \neq 0 \) given log weight is in the model. With a \( p \)-value < 0.0001 < 0.01 = \( \alpha \) we reject \( H_0 \) and conclude litter size is a significant predictor of log brain weight even after accounting for log weight.

e) Type III test is \( H_0: B_{\text{litter}} = 0 \) given log weight & gest are in the model vs. \( H_1: B_{\text{litter}} \neq 0 \) given log weight & gest are in the model with \( p \)-value = 0.0008 < 0.01 = \( \alpha \) we reject \( H_0 \). Litter size is a significant predictor, even accounting for log weight and gestation period.
6a) \( e^{0.0262} = 1.3 \) The odds of success increase by 1.3 for each extra 10 minutes.

b) \( (e^{0.0005}, e^{0.0518}) = (1.00050, 1.053165) \). The odds of success increase by .050% to 5.3% for each additional minute.

c) \( H_0: B_1 \leq 0 \)
\( H_A: B_1 > 0 \)
\[ Z = \frac{0.0262}{0.0131} = 2 \]
Fail to reject

Critical Value: \( Z_{0.01} = 2.33 \).

d) \[ prob = \frac{1}{1 + \exp(- (B_0 + B_1 60) )} = \frac{1}{1 + \exp(-1.0133 - 0.0262 \times 60) } \]
\[ = \frac{1}{1 + \exp(-2.5853)} = \frac{1}{1 + 0.7537} = .9299 \]