Syllabus
Analysis Qualifying Exam

These are the topics to be covered for preparation for the Qualifying Exam portion in Analysis. It is generally expected that Instructors in Math 703-704 will cover the majority of these topics in some detail.

Lebesgue theory of Measure and Integration

- Metric spaces, compactness, continuous functions and Weierstrass approximation theorem.
- Outer measure, measurable sets, measure spaces, complete and regular measures.
- Integration, Fatou's lemma and convergence theorems
- The Extension Theorem, product measure and Fubini's theorem,
- Lebesgue-Stieltjes integral
- Absolute continuity, Vitali's lemma, differentiation theory for monotone functions and integrals, functions of bounded variation.
- Egorov and Lusin Theorems
- Definition of $L^p$, Hölder and Minkowski inequalities, completeness of $L^p$, approximation by step and continuous functions.

Complex Analysis

- Analytic Functions: complex derivatives and Cauchy-Riemann equations, analyticity, special functions: $\log(z)$, $e^z$, trig functions.
- Conformal mappings and linear fractional transformations.
- Line integrals, Cauchy's theorem and its consequences:
- Cauchy integral formula, maximum modulus, power series,
- Fundamental Theorem of Algebra.
- Classification of zeros and singularities, Laurent series,
- Argument principle.
- Residue theorem, evaluation of integrals and series.

Reference Texts:
0. Elias M. Stein & Rami Shakarchi, Real Analysis, Princeton Lectures in Analysis II, 2005
1. W. Ruckle, Modern Analysis, PSW-Kent, Boston, 1991