State High School Mathematics Tournament

University of South Carolina

February 3, 2018
Tiebreaker Rules

▶ You will be asked one question whose answer is a positive integer, and you will have 60 seconds to answer.

▶ Solving it exactly within 60 seconds is probably impossible.

▶ Try to solve it approximately, as accurately as you can, and make an educated guess.

▶ The answer(s) closest to the truth (in either direction) win the tiebreaker.
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Tiebreaker Question

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- The integer \(m^2 + n^2\) is one plus an integer multiple of 4.
- We have \(m^2 + n^2 \leq 2018\).
Answer. 3176 (ask a computer).
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- Pairs \((m, n)\) with \(m^2 + n^2 \leq 2018\) correspond to lattice points inside (or on) the circle \(m^2 + n^2 \leq 2018\).
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- Pairs \((m, n)\) with \(m^2 + n^2 \leq 2018\) correspond to lattice points inside (or on) the circle \(m^2 + n^2 \leq 2018\).
- The number of such lattice points is approximately the area of the circle, \(2018\pi\).
Tiebreaker Answer

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- The number of such lattice points is approximately the area of the circle, \(2018\pi\).
- \(m^2 + n^2\) will be one plus an integer multiple of 4 if and only if \(m\) and \(n\) are of opposite signs, so we should count only half the lattice points.

Doing better requires brute force or a computer.

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- So the answer is approximately \(1009\pi = 3169.86\ldots\)
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- So the answer is approximately

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